Linear programming as a tool to study the stability of masonry arch bridges

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Summary

By Linear Programming it is possible to get the actual collapse load factor of the Limit Analysis, and also enables the optimization of other functions to study the behaviour of masonry structures. In this article two examples of application of the effect of the backfill weight on masonry arches are shown.

Keywords: masonry arches, backfill, limit analysis, linear programming.

1. Introduction

Even today the safety assessment of the historic masonry structures represents a problem [1]. In the previous steps to the collapse, it is not easy to obtain the actual state of stress and, in most cases, the boundary conditions are not fully certain. Generally the material is heterogeneous and anisotropic, presenting highly compressive strength, low tensile strength and brittle fracture. However, it is well established that collapse occurs far from the first crack appearance. Generally, the failure is caused by instability when a sufficient number of plastic hinges transform the structure in a mechanism. In cases where no slip occurs [2,3,4], many authors [5,6,7] have proposed the application of the Limit Analysis theorems since they constitute an excellent simplified tool. However, some important questions remain unknown: "A priori" it is not obvious to evaluate if an action induces stability or instability effects and, therefore, neither are the criteria about weighting loads that should be used.

2. Linear Programming and Limit Analysis

2.1 The static theorem as a linear program: formulation and extensions

This work has two main purposes. First, to show what linear programming is and how it can implement the formulation of static limit theorems. On the other hand, this work shows how to expand the formulation in order to allow the exploration of the range of stable solutions for masonry arch bridges, before using other more sophisticated analysis tools.

Briefly, linear programming (LP) is a method to find the maximum or the minimum of a linear function subjected to linear equality and inequality constraints. Let $\mathbf{B}^t$ the equilibrium matrix, $\mathbf{L}^t$ the yield matrix, **s** the vector of stresses, **y** a vector of positive slack variables, **f** the vector of loads, **g** of dead loads, **q** of live loads and l a scalar load factor, all admissible static solutions are subject to the constraints (1).

$$\mathbf{B}^t\mathbf{s} = \mathbf{f}\ ;\ \mathbf{L}^t\mathbf{s} = \mathbf{y} \geq \mathbf{0}\ ;\ \mathbf{f} = \mathbf{g} + l\mathbf{q} + \dots \quad (1)$$

Since theorems of Standard Limit Analysis can be formulated in this way, the LP allows to obtain an upper limit of load factor l corresponding to its static formulation. Further details about implementation are described elsewhere [4,8]. This factor is the actual load of the onset of collapse corresponding to the uniqueness theorem. The equivalence between the Limit Analysis theorems

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